Notes and Examples for Part 4:

- Trigonometry
- Law of Sines
- Law of Cosines

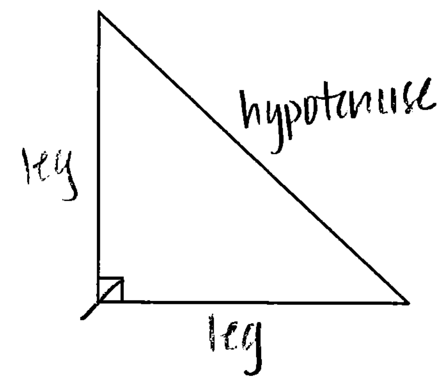
PreCalculus
Notes – Unit 4

Name:____________________
Date: ____________________

Introduction to Trigonometry: Essential Concepts for Solving Right Triangles

I. Review of Trigonometry

- In a right triangle, one of the angles measures $90°$ and the remaining two angles are acute and complmentary.
- The longest side of the right triangle is called the hypotenuse and is always opposite the right angle. The other two sides of the triangle are called legs.



- In Geometry, three trigonometric ratios are discussed. They are:

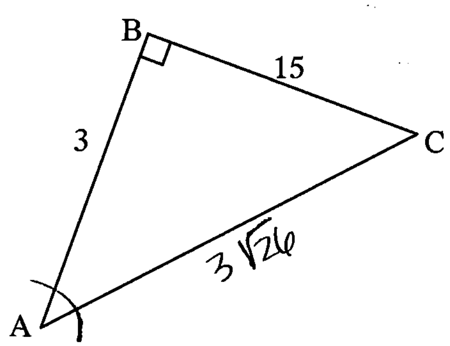
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

- Another key concept that is necessary when solving right triangles is the Pythagorean Theorem, which is $a^2+b^2=c^2$.

no decimals!! exact answers only!!

II. Examples

1. Find the values of sine, cosine and tangent for $\angle A$. Leave the answer in simplest fraction form and/or simplest radical form.



$3^2+15^2=c^2$
$9+225=c^2$
$\sqrt{234}=\sqrt{c^2}$
9 26
$3\sqrt{26}=c$

$\tan A = \dfrac{15}{3} = \dfrac{5}{1}$

$\sin A = \dfrac{15}{3\sqrt{26}} = \dfrac{5}{\sqrt{26}} = \dfrac{5\sqrt{26}}{26}$

$\cos A = \dfrac{3}{3\sqrt{26}} = \dfrac{1}{\sqrt{26}} = \dfrac{\sqrt{26}}{26}$

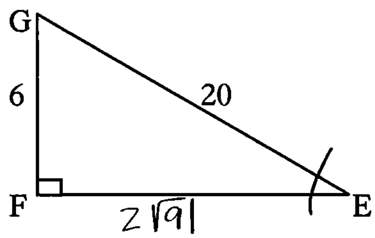
III. More Trigonometric Ratios

Sine, cosine and tangent functions have reciprocals. They are:

- The reciprocal of sine is cosecant. The ratio is $\frac{\text{hypotenuse}}{\text{opposite}}$.
- The reciprocal of cosine is secant. The ratio is $\frac{\text{hypotenuse}}{\text{adjacent}}$.
- The reciprocal of tangent is cotangent. The ratio is $\frac{\text{adjacent}}{\text{opposite}}$.

IV. Examples

1. Find the values of the six trigonometric ratios for $\angle E$.



$$6^2 + b^2 = 20^2$$
$$36 + b^2 = 400$$
$$b^2 = 364 \;<\; \begin{matrix}4\\91\end{matrix}$$
$$b = 2\sqrt{91}$$

$$\sin E = \frac{6}{20} = \boxed{\frac{3}{10}}$$

$$\cos E = \frac{2\sqrt{91}}{20} = \boxed{\frac{\sqrt{91}}{10}}$$

$$\tan E = \frac{6}{2\sqrt{91}} \cdot \frac{\sqrt{91}}{\sqrt{91}} = \frac{6\sqrt{91}}{182} = \boxed{\frac{3\sqrt{91}}{91}}$$

$$\csc E = \boxed{\frac{10}{3}}$$

$$\sec E = \frac{10}{\sqrt{91}} = \boxed{\frac{10\sqrt{91}}{91}}$$

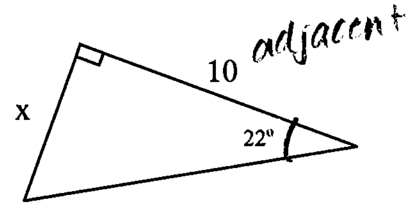
$$\cot E = \frac{2\sqrt{91}}{6} = \boxed{\frac{\sqrt{91}}{3}}$$

V. Solving ALL Parts of the Triangle

We can use trigonometric ratios to solve for angles, as well as sides!

* make sure your calculator is in degrees *

1.



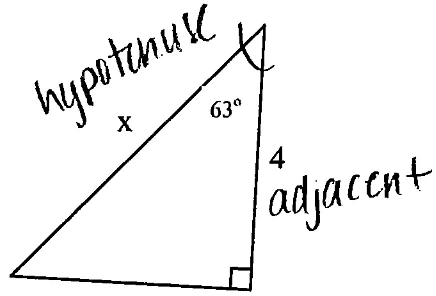
use tangent

$$\tan 22 = \frac{x}{10}$$

$$10(\tan 22) = x$$

$$\boxed{4.04 = x}$$

2.



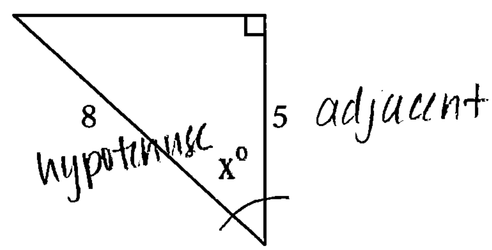
use cosine!

$$\cos 63 = \frac{4}{x}$$

$$x = \frac{4}{\cos 63}$$

$$\boxed{x = 8.81}$$

3.



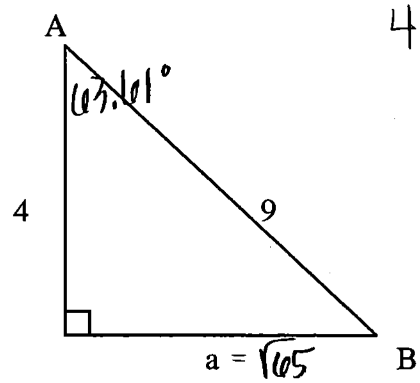
use cosine!

$$\cos x = \frac{5}{8}$$

$$x = \cos^{-1}\left(\frac{5}{8}\right)$$

$$\boxed{x = 51.32°}$$

4. Solve for all unknown parts of the triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



$$4^2 + a^2 = 9^2$$

$$a^2 = 65$$

$$\boxed{a = \sqrt{65}}$$

$$\sin A = \frac{\sqrt{65}}{9}$$

$$A = \sin^{-1}\left(\frac{\sqrt{65}}{9}\right) = \boxed{63.61° = A}$$

$$180 - (90 + 63.61)$$

$$\boxed{B = 26.39°}$$

Homework: Practice 13-1 Worksheet Odds

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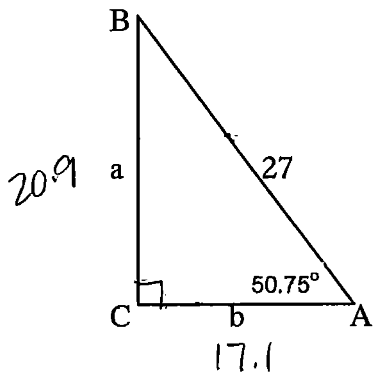
More Introduction to Trigonometry: Right Triangle Applications

1. Solve ΔABC by using the measurements given. $\angle C = 90°$. Round side measures to the nearest tenth.



$\sin 50.75 = \frac{a}{27}$

$27 \sin 50.75 = a$

$\boxed{20.9 = a}$

$20.9^2 + b^2 = 27^2$

$b^2 = 292.19$

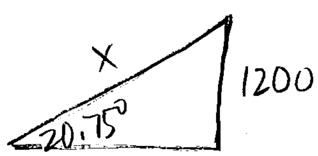
$\boxed{b = 17.1}$

$B = 180 - (50.75 + 90)$

$\boxed{B = 39.25°}$

* draw a diagram *

2. A chair lift at a ski resort rises at an ***angle of elevation*** of $20.75°$ and attains a vertical height of 1200 feet. How far does the chair lift travel up the side of the mountain? Round to the nearest tenth.

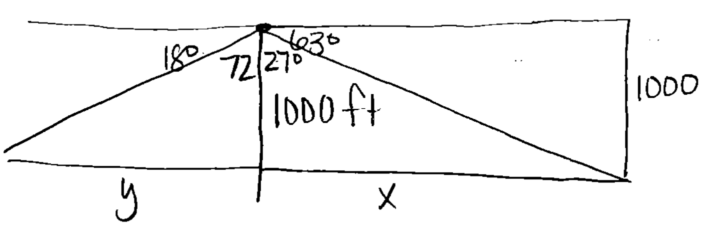


$\sin 20.75 = \frac{1200}{x}$

$x = \frac{1200}{\sin 20.75}$

$\boxed{x = 3387.0 \text{ ft}}$ * label units *

3. A traffic helicopter is flying 1000 feet above the downtown area. To the right, the pilot sees the baseball stadium at an ***angle of depression*** of $63°$. To the left, the pilot sees the football stadium at an angle of depression of $18°$. Find the distance between the two stadiums. Round to the nearest tenth.



$\tan 27 = \frac{x}{1000}$

$1000 \tan 27 = x$

$509.5 = x$

$\tan 72 = \frac{y}{1000}$

$1000 \tan 72 = y$

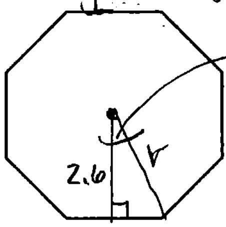
$3077.7 = y$

total

$\boxed{3587.2 \text{ ft}}$

4. The apothem of a regular polygon is the measure of a line segment from the center of the polygon to the midpoint of one of its sides. The apothem of a regular octagon is 2.6 cm. Find the radius of the circle circumscribed about the octagon. Round to the nearest tenth.

make a right triangle. 8 sides → 16 slices



$\frac{360}{\#\text{slices}} = \frac{360}{16} = 22.5°$

$\cos 22.5 = \frac{2.6}{r}$

$r = \frac{2.6}{\cos 22.5}$

$\boxed{r = 2.8}$

Find the area $= \frac{1}{2}ap$

$\tan 22.5 = \frac{p}{2.6}$

$2.6 \tan 22.5 = p$

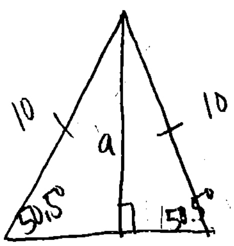
$1.1 = p$

$1.1 \cdot 16 = p$

$17.6 = p$

$A = \frac{1}{2}ap = \frac{1}{2}(2.6)(17.6) = \boxed{22.9\ cm^2}$

5. Each base angle of an isosceles triangle measures $50.5°$. Each of the congruent sides is 10 cm long.



a. Find the altitude of the triangle. Round to the nearest tenth.

$\sin 50.5 = \frac{a}{10}$

$10 \sin 50.5 = a$

$\boxed{7.7\ cm = a}$

b. What is the length of the base? Round to the nearest tenth.

$\cos 50.5 = \frac{b}{10}$

$10 \cos 50.5 = b$

$6.4 = b$

$6.4 \times 2 = \boxed{12.8\ cm}$

c. Find the area of the triangle. Round to the nearest tenth.

$A = \frac{1}{2}bh = \frac{1}{2}(12.8)(7.7) = \boxed{49.3\ cm^2}$

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Unit 4: Solving Triangles Using Law of Sines

I. Law of Sines Equation

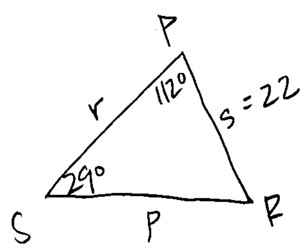
a. The Law of Sines can be used to solve triangles that are NOT right triangles.

b. Let ΔABC be any triangle with a, b, and c representing the measures of the sides opposite angles with measures A, B, and C respectively. Then the following is true:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

II. Examples

a. Solve ΔSPR if $S = 29°$, $P = 112°$ and s = 22. Draw a diagram to help you solve. Round to the nearest tenth.



① Find the measure of the 3rd angle

$180 - (29 + 112) = \boxed{39° = R}$

② use Law of Sines

$\frac{22}{\sin 29} = \frac{r}{\sin 39}$

$r = \frac{22 \sin 39}{\sin 29}$

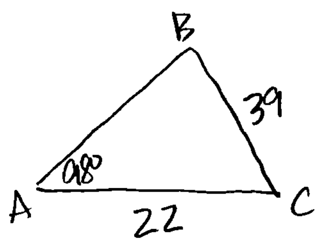
$\boxed{r = 28.6}$

$\frac{22}{\sin 29} = \frac{p}{\sin 112}$

$p = \frac{22 \sin 112}{\sin 29}$

$\boxed{p = 42.1}$

b. Solve ΔABC if $A = 98°$, a = 39 and b = 22. . Draw a diagram to help you solve. Round to the nearest tenth.



$\frac{39}{\sin 98} = \frac{22}{\sin B}$

$\sin B = \frac{22 \sin 98}{39}$

$B = \sin^{-1}\left(\frac{22 \sin 98}{39}\right)$

$\boxed{B = 34°}$

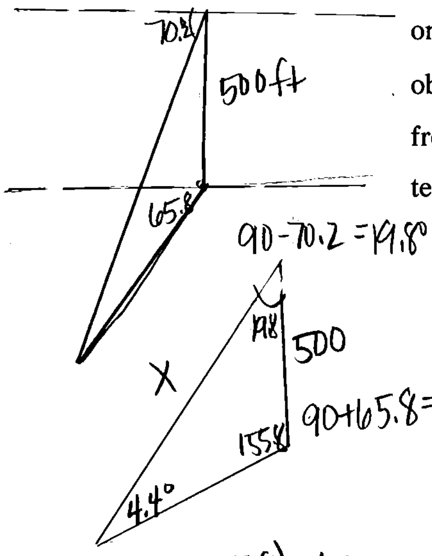
$180 - (98 + 34) = \boxed{48° = C}$

$\frac{c}{\sin 48} = \frac{39}{\sin 98}$

$c = \frac{39 \sin 48}{\sin 98}$

$\boxed{c = 29.3}$

c. A person in a hot air balloon observes that the angle of depression to a building on the ground is 65.~~9~~8°. After ~~descending~~ ascending vertically 500 feet, the person now observes that the angle of depression is 70.2°. How far is the balloonist now from the building? Draw a diagram to help you solve. Round to the nearest tenth.



$90 - 70.2 = 19.8°$

$90 + 65.8 = 155.8$

$180 - (19.8 + 155.8) = 4.4°$

$$\frac{500}{\sin 4.4} = \frac{X}{\sin 155.8}$$

$$\frac{500 \sin 155.8}{\sin 4.4} = X$$

$$\boxed{2671.6\text{ ft} = X}$$

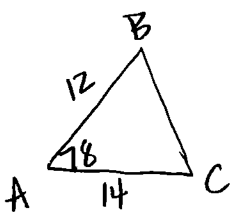
III. Finding Area using Law of Sines

a. Let ΔABC be any triangle with a, b, and c representing the measures of the sides opposite angles with measures A, B, and C respectively. Then the area K can be determined by using one of the following formulas.

$$K = \frac{1}{2}bc\sin A \qquad K = \frac{1}{2}ab\sin C \qquad K = \frac{1}{2}ac\sin B$$

IV. Area Examples – Find the area of each triangle. Round to the nearest tenth.

a. $A = 78°$, b = 14, c = 12

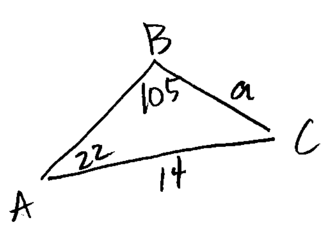


$K = \frac{1}{2} bc \sin A$

$K = \frac{1}{2}(12)(14)\sin(78) = \boxed{82.2\ u^2}$

b. $A = 22°$, $B = 105°$, b = 14

need 2 sides first - use Law of Sines...



$180 - (22 + 105)$

$\boxed{53° = C}$

$$\frac{14}{\sin 105} = \frac{a}{\sin 22}$$

$$\frac{14 \sin 22}{\sin 105} = a$$

$$\boxed{5.4 = a}$$

$K = \frac{1}{2} ab \sin C$

$K = \frac{1}{2}(5.4)(14)\sin 53$

$= \boxed{30.2\ u^2}$

Practice Problems: Page 316 – 317, 12 – 24 even, 32 – 34 all (everyother written above "even")

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Unit 4: Law of Cosines

I. Law of Cosines

a. The Law of Cosines applies to triangles where <u>two</u> sides of a triangle are known, along with an included angle, or when you know only sides!

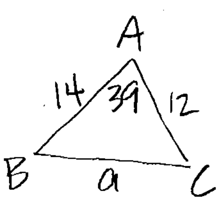
b. The Law of Cosines states: Let ΔABC be any triangle with a, b and c representing the measures of sides opposite angles with measures A, B and C respectively. Then the following are true:

1. $a^2 = b^2 + c^2 - 2bc\cos A$
2. $b^2 = a^2 + c^2 - 2ac\cos B$
3. $c^2 = a^2 + b^2 - 2ab\cos C$

****watch order of operations****

II. Examples

Use Law of Cosines to solve each triangle. You may want to draw a picture. Round to the nearest tenth.



1. $A = 39°$, b = 12, c = 14.

$a^2 = 14^2 + 12^2 - 2(14)(12)\cos 39$

$a^2 = 340 - 261.1$

$a^2 = 78.9$

$\boxed{a = 8.9}$

now, use Law of Sines to finish...

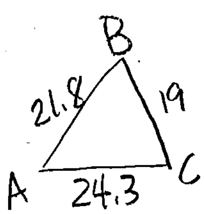
$\frac{8.9}{\sin 39} = \frac{12}{\sin B}$

$\sin B = \frac{12\sin 39}{8.9}$

$\boxed{B = 58.1°}$

$180 - (39 + 58.1)$

$= \boxed{82.9° = C}$



2. a = 19, b = 24.3, c = 21.8

$a^2 = b^2 + c^2 - 2bc\cos A$

$19^2 = 24.3^2 + 21.8^2 - 2(24.3)(21.8)\boxed{\cos A}$ ~ solve for A!

$361 = 1065.7 - 1059.5\cos A$

$-1065.7 \quad -1065.7$

$\frac{-704.7}{-1059.5} = \frac{-1059.5\cos A}{-1059.5}$

$0.6651 = \cos A$

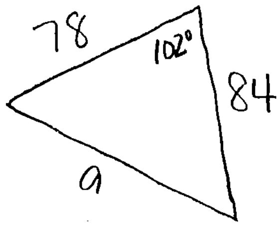
$\boxed{48.3° = A}$

$\frac{19}{\sin 48.3} = \frac{24.3}{\sin B}$

$\boxed{B = 72.7°}$

$180 - (72.7 + 48.3) = \boxed{59° = C}$

3. Suppose that you want to fence a triangular lot. If two sides measure 84 feet and 78 feet, and the angle between them is $102°$, what is the length of the fence? Round to the nearest foot.

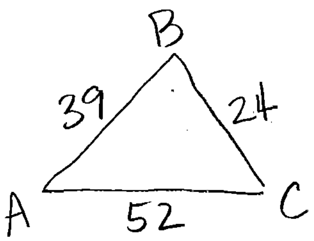


$a^2 = 78^2 + 84^2 - 2(78)(84)\cos 102$

$a^2 = 15864.5$

$a = 126$ ft

perimeter = $\boxed{288 \text{ ft}}$



4. Find the area of ΔABC if a = 24, b = 52 and c = 39.

Use $K = \frac{1}{2}ab\sin C$ ... so we need $\angle C$.

$39^2 = 24^2 + 52^2 - 2(24)(52)\cos C$

$1521 = 3280 - 2496\cos C$

$-3280 \quad -3280$

$\frac{-1759}{-2496} = \frac{-2496\cos C}{-2496}$

$0.7047 = \cos C$

$\boxed{45.2° = C}$

$K = \frac{1}{2}(24)(52)\sin 45.2 = \boxed{442.8 \text{ units}^2}$

Practice Problems on Page 330-331
Mandatory Problems: 6 – 24 even
Suggested Problems: 11 – 23 odd